R+D 6036-MA-01.

RMCS
Royal Military College of Science
Shrivenham
Swindon Wilts SN6 8LA England
Telephone: Army Network: SHRIVENHAM MILITARY
Civil Network: (0793) 782551 Extension
Telex: 265871 Ref WJJ110
Fax: (0793) 783878

Cranfield

(2)

DEVELOPMENT OF A COMBAT MODEL WITH A MINIBATTLE STRUCTURE

by

Michael R. Bathe
Kenneth R. McNaught
Sean R. Belton

This is the second Interim Report on work supported by the US Army Research, Development and Standardization Group - UK under contract number DAJA45-88-C-0031.

SYSTEMS ASSESSMENT GROUP JULY 1989

Development of a Combat Model with a Minibattle Structure

1. This is the second Interim Report on a study to develop a prototype model of combat based on the decomposition of a large scale battle into a number of miibattles.

2. Further analysis of combat trials data has been carried out and some of the results of that analysis will be incorporated into our prototype combat model.

Data Analysis

3. The results presented here were obtained by analysing data from the ARCOMS series of armoured combat trials held at Fort Hood, Texas. A total of twenty-four separate battles were studied.

4. An area of particular interest to us was the distribution of activity among weapon systems. We were interested both in how the involvement of individual weapon systems varied and in how the level of total activity displayed by each side varied as the battle progressed.

5. The level of activity or involvement that a weapon system displays during a battle can be defined in more than one way. A very strict definition might only include the time spent engaging enemy weapons ie. the time spent firing and preparing to fire. A less strict definition might also include the time spent trying to detect an enemy weapon to fire at. Less strict still, if it is really the level of a weapon system's involvement in a battle that is of interest then surely the time that a weapon spends under attack, ie. being fired at, must be included as well. Consequently, we have two different types of involvement (or activity) which shall be defined as follows.

   a) aggressive activity, where the weapon system in question is firing or preparing to fire

   b) passive activity, where the weapon system in question is simply a target ie. it is being engaged by an enemy weapon but is not firing itself.

6. Naturally, the case will often arise where a weapon system is both firing and coming under fire at the same time. For our purposes, however, such a situation has been included in the category of aggressive activity.

7. Having now defined what we mean by activity, then, Figure 1 shows the variations in the mean number of weapon systems on each side involved in aggressive activity as functions of elapsed battle time. These figures, and all the other figures presented here unless otherwise stated, have been averaged over the twenty-four separate battles.

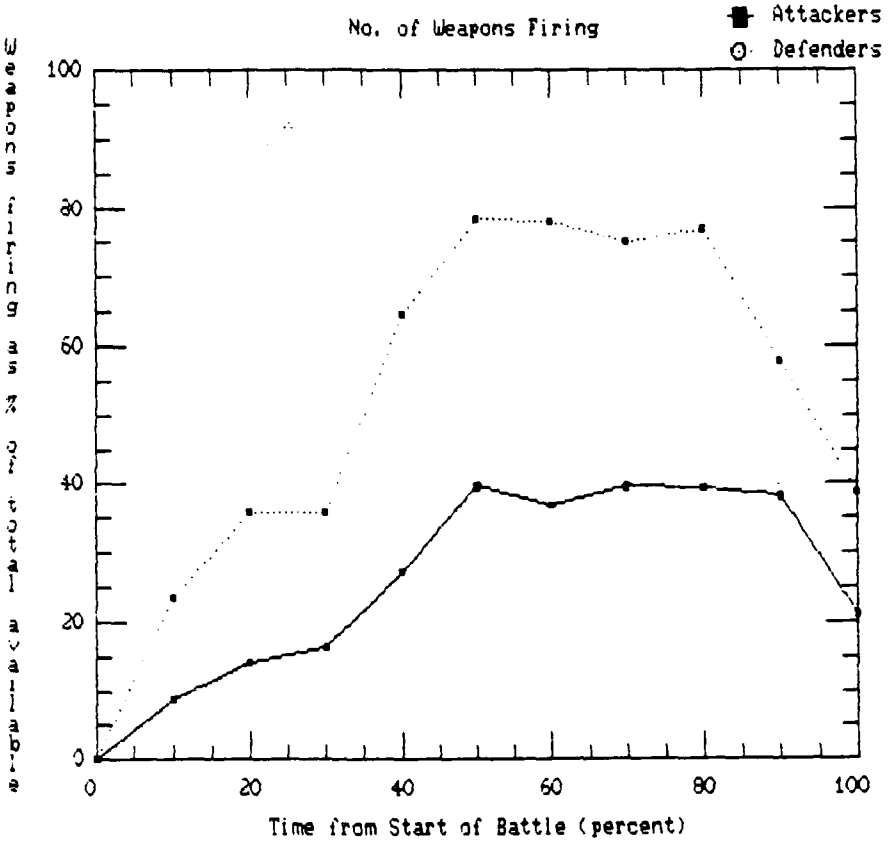


Figure 1

8. In order to find comparable results from each of the battles analysed, the fact that each battle lasted for a different length of time had to be taken into account. This was achieved by splitting each battle up into ten time zones covering the entire battle from first shot to last. Within each battle, the time zones are of equal length but obviously this length varies from battle to battle. Hence, the X-axis on the above graph shows elapsed battle time as a percentage rather than as an explicit time.

9. The graph shows the number of weapon systems involved in aggressive activity as a percentage of the total number available at that time ie. for each time zone, the number of weapons firing at least one shot in that time zone was recorded for each battle and these numbers were then summed over all the battles and divided by the total number of survivors in that time zone to give the values displayed.

10. In order to resolve queries concerning the proportion of available time that is actually spent engaging the enemy at various stages of a battle, Figure 2 shows the fraction of total time spent in the aggressively active state as a function of elapsed battle time. These results take into account the lengths of engagement sequences unlike those in Figure 1 which treat all firing events occurring in the same time zone the same regardless of their length. Like Figure 1, Figure 2 also takes account of the varying number of survivors when calculating the total time available for action in each of the time zones.

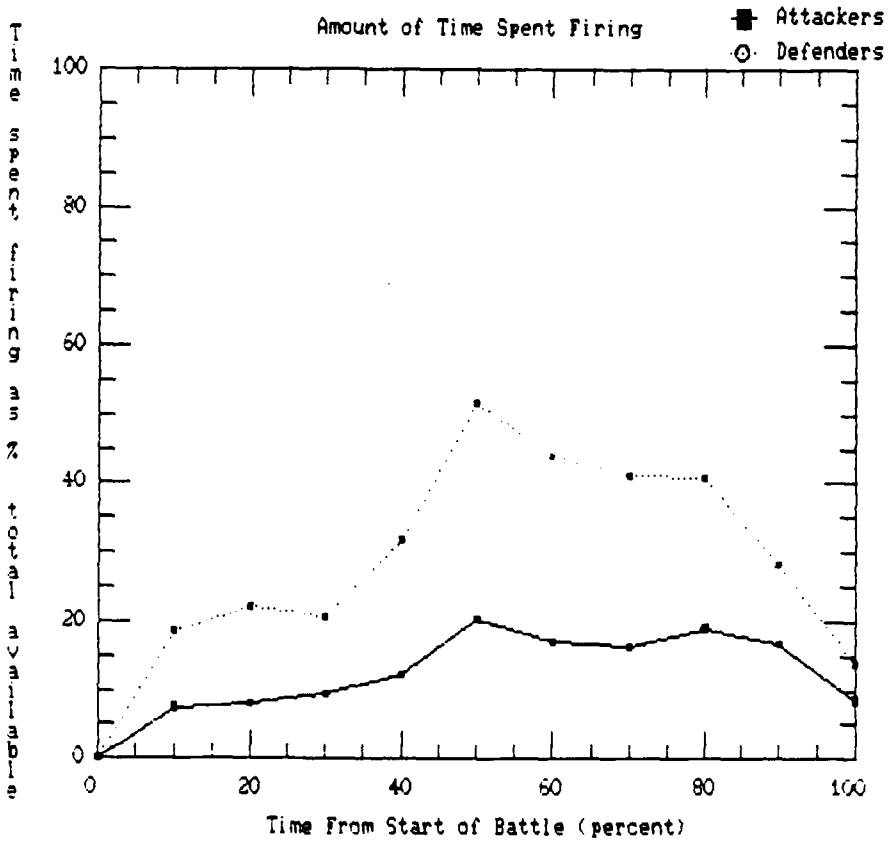


Figure2

11. Figure 3 shown below corresponds to Figure 1 in that the values plotted are the numbers of weapons actively involved as proportions of the total number of weapons available in each time zone. However, the definition of activity has now been broadened to include passive activity as previously defined ie. the time spent being shot at without returning fire.

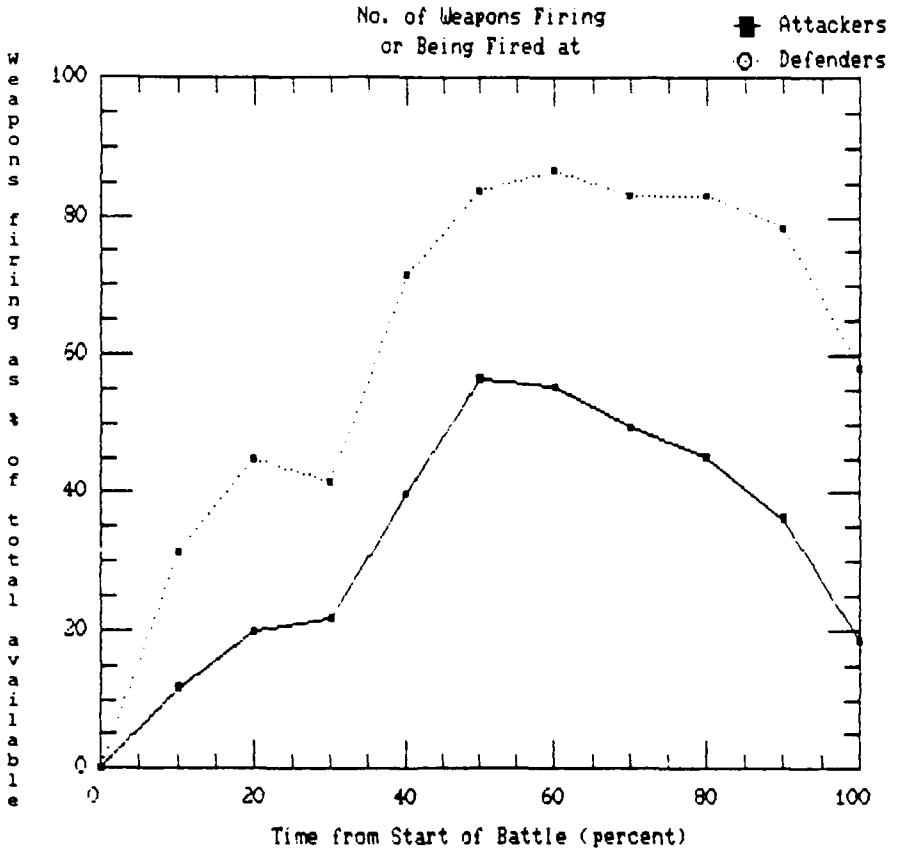


Figure 3

12. Likewise, Figure 4 corresponds to Figure 2 in that the values plotted show the time spent in an active state as a proportion of the total time available in each time zone but again the definition of activity has been changed to include passive activity as well.

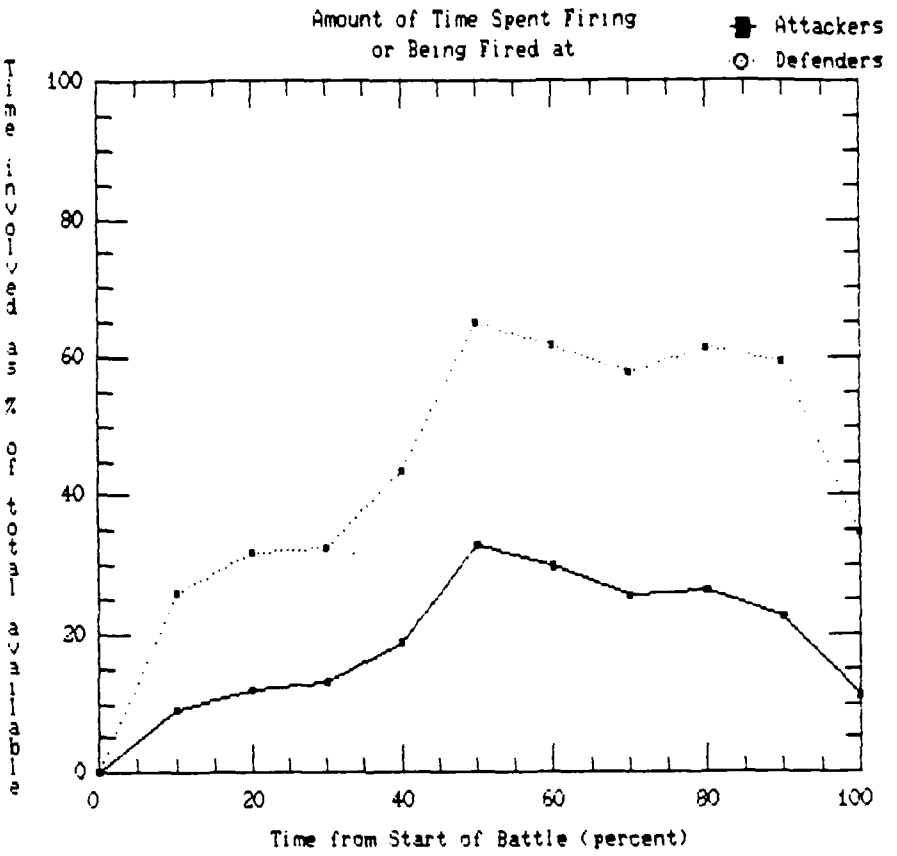


Figure 4

13. Now turning our attention to the total time spent in an active state by an individual weapon system during a battle, Figures 5 and 6 show the distributions of the total time in seconds that individual weapon systems spent aggressively involved in a battle. Figure 5 corresponds to attacking forces and Figure 6 to defending forces.

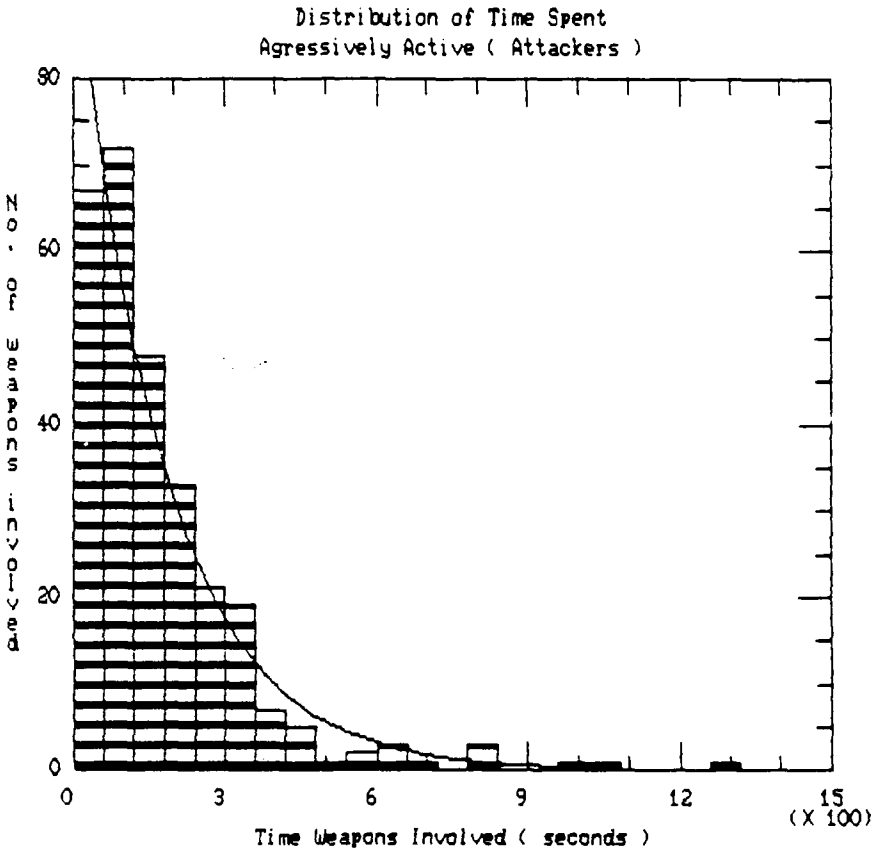

Distribution of Time Spent
Agressively Active (Attackers)
No. of weapons involved
80
60
40
20
0
0
3
6
9
12
15
(X 100)
Time Weapons Involved (seconds)

Figure 5

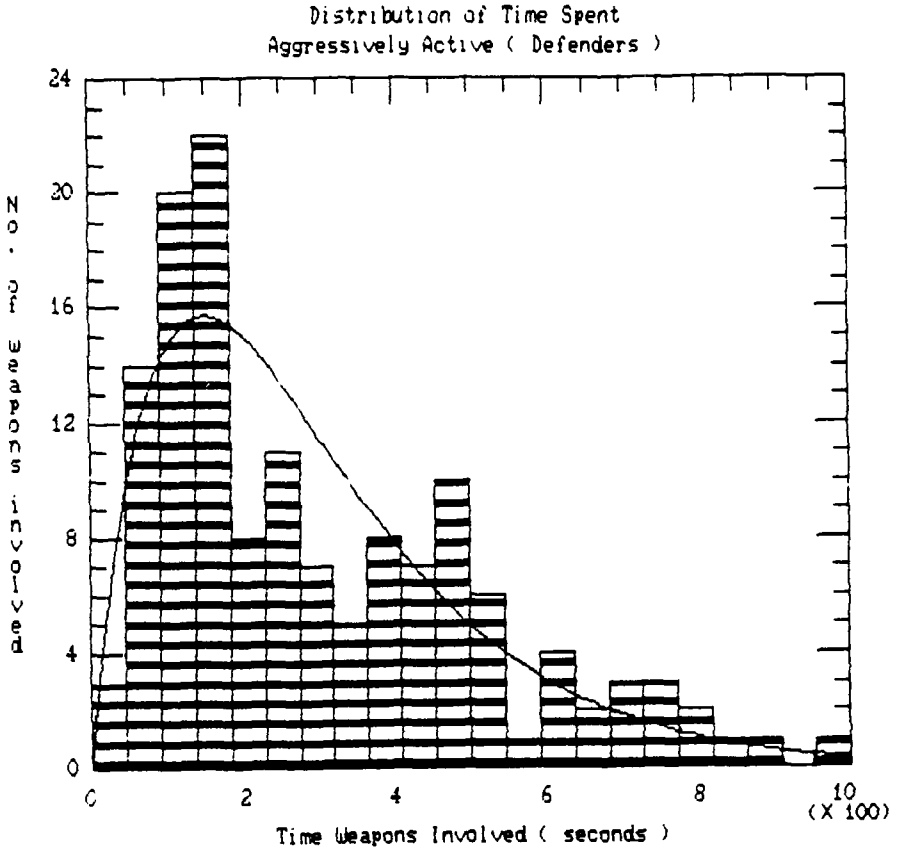


Figure 6

14. The distribution of time spent in the aggressive state by individual attackers appeared to follow the negative exponential distribution with a mean of 175 seconds. The defenders' time distribution, however, seemed to follow the two-stage Erlang distribution with a mean of 297 seconds. Overlaid plots of these probability distributions appear on the graphs for comparison. The chi-squared tests shown in Figures 7a and 7b confirm the goodness of fit given by these distributions at the 5% level.

Chisquare Test

	Lower Limit	Upper Limit	Observed Frequency	Expected Frequency	Chisquare
	at or below	60.00	67	83	3.0529410
	60.00	120.00	72	59	2.9680940
	120.00	180.00	48	42	.9558438
	180.00	240.00	33	30	.4002787
	240.00	300.00	21	21	.0000733
	300.00	360.00	19	15	1.1514597
	360.00	420.00	7	11	1.1885280
	420.00	480.00	5	7	.8185161
	480.00	540.00	1	5	3.4879072
	540.00	660.00	5	6	.3148986
above	660.00		7	6	.0393313

Chisquare = 14.3779 with 9 d.f. Sig. level = 0.109504

Figure 7a

Chisquare Test

	Lower Limit	Upper Limit	Observed Frequency	Expected Frequency	Chisquare
	at or below	45.45	4	5	.34501
	45.45	90.91	14	12	.24572
	90.91	136.36	20	15	1.54865
	136.36	181.82	22	16	2.55719
	181.82	227.27	8	15	3.16667
	227.27	272.73	11	13	.42584
	272.73	318.18	7	12	1.86177
	318.18	363.64	5	10	2.43370
	363.64	409.09	8	8	.00913
	409.09	454.55	7	7	.00517
	454.55	500.00	10	6	3.57736
	500.00	590.91	7	8	.13818
	590.91	681.82	6	5	.15810
above	681.82		11	8	1.17637

Chisquare = 17.6489 with 11 d.f. Sig. level = 0.090094

Figure 7b

15. Broadening our definition of activity again to include aggressive and passive activity, Figures 8 and 9 show the new distributions that result. Figure 8 corresponds to attackers and Figure 9 to defenders.

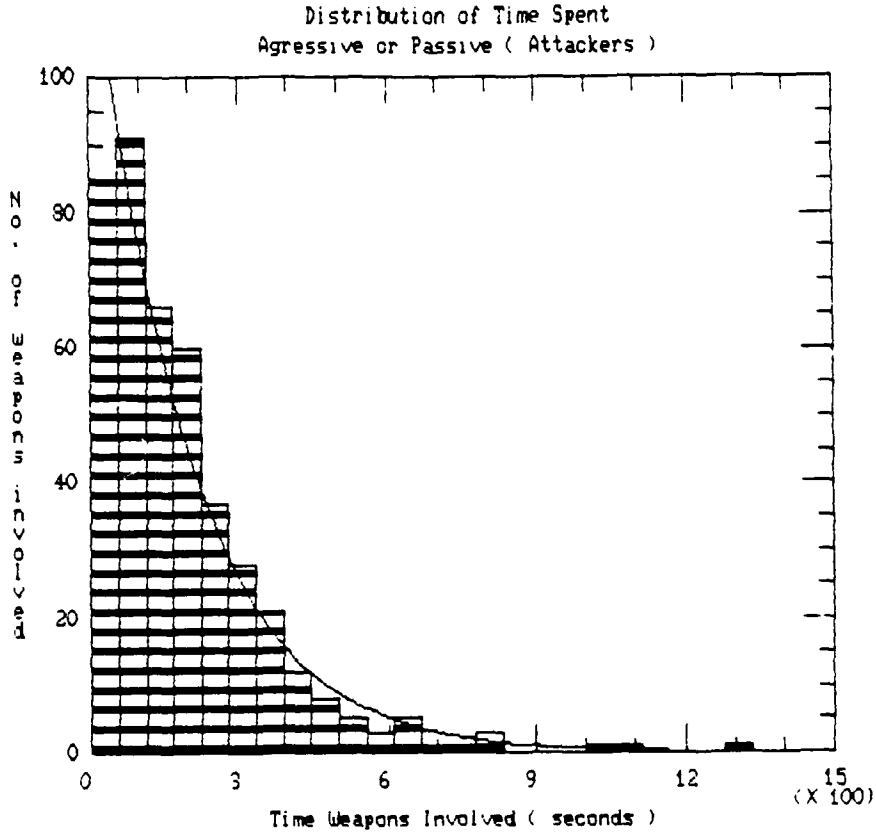


Figure 8

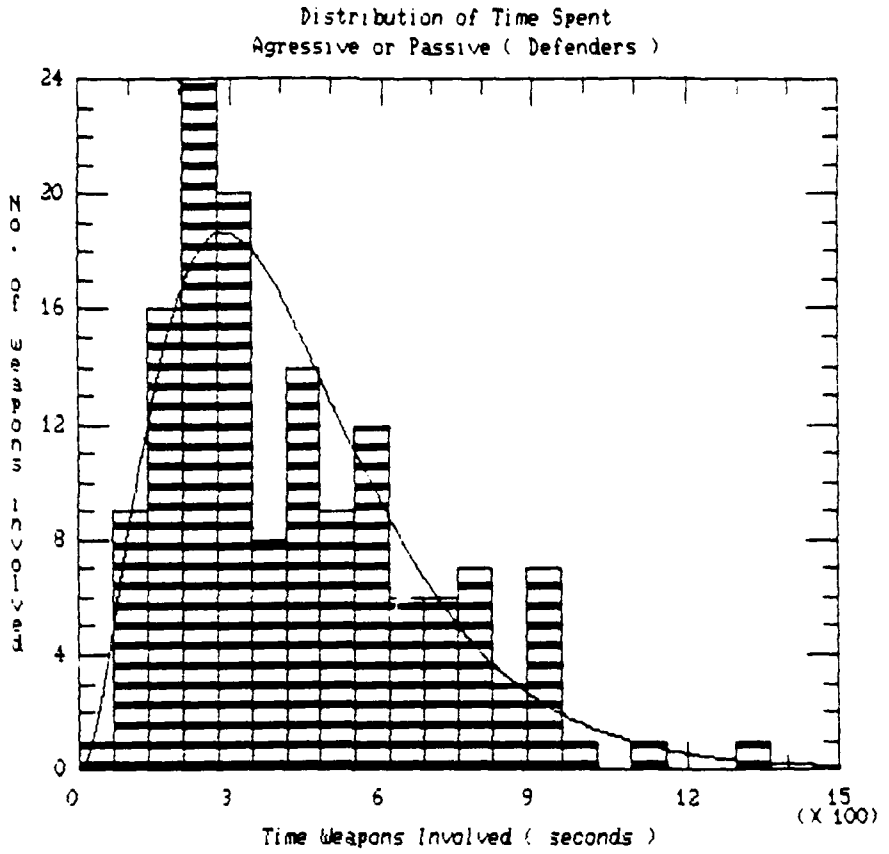


Figure 9

16. The distribution of time spent either aggressively or passively active by attackers was still approximated by the negative exponential distribution - this time with a mean of 187 seconds. The new results for the defenders, however, were now fitted by a three-stage Erlang distribution, reflecting the generally greater level of involvement expected. The mean value this time was 430 seconds. Again, overlaid plots of these probability distributions appear on the graphs for comparison and the corresponding chi-squared tests are shown in Figures 10a and 10b. They confirm the goodness of fit of these distributions to the results at the 5% significance level.

Chisquare Test

	Lower Limit	Upper Limit	Observed Frequency	Expected Frequency	Chisquare
	at or below	55.56	85	110	5.800
	55.56	111.11	91	82	1.002
	111.11	166.67	66	61	.432
	166.67	222.22	60	45	4.829
	222.22	277.78	37	34	.345
	277.78	333.33	28	25	.371
	333.33	388.89	21	19	.326
	388.89	444.44	12	14	.229
	444.44	500.00	8	10	.487
	500.00	555.56	5	8	.891
	555.56	611.11	3	6	1.241
	611.11	722.22	6	7	.236
above	722.22		7	9	.447

Chisquare = 16.637 with 11 d.f. Sig. level = 0.119082

Figure 10a

Chisquare Test

	Lower Limit	Upper Limit	Observed Frequency	Expected Frequency	Chisquare
	at or below	136.36	10	10	.01487
	136.36	204.55	16	15	.10889
	204.55	272.73	24	18	2.01102
	272.73	340.91	20	19	.11908
	340.91	409.09	8	17	4.93303
	409.09	477.27	14	15	.06161
	477.27	545.45	9	12	.92673
	545.45	613.64	12	10	.44782
	613.64	681.82	6	8	.36942
	681.82	750.00	6	6	.00460
	750.00	886.36	10	8	.79684
above	886.36		10	8	.59201

Chisquare = 10.3809 with 9 d.f. Sig. level = 0.320156

Figure 10b

Model Development

17. The prototype model has been set up to read from three separate input files which will hold the data on weapon system characteristics, force sizes, deployments, etc. This data will have been entered at the keyboard by the user answering a series of displayed questions. The three files correspond to three distinct types of information :-

    a) information regarding weapon systems' characteristics

    b) information regarding the composition of weapon groups

    c) information regarding force deployments.

18. This division is intended to make it easier to change or correct data entered previously.

19. It has also been decided to give the forces on each side a group structure in order to reduce the amount of data entry required. Groups will be composed of weapons of the same type which will travel together along the same route. A typical attacking group might be a tank company whereas a defending group might comprise a platoon, a section or an individual weapon system.

20. The attacking forces will follow a set of attack paths entered by the user while the defending forces will occupy a set of static positions. This will make it easier to identify potential engagements.

Attrition Methodologies

21. As previously discussed [A], there are several possible methods of resolving attrition within the minibattles and the intention is to experiment with these until satisfactory results are obtained in terms of speed and realism.

22. The stochastic duel model ''MATADOR'' [B,C], developed by colleagues at RMCS, has been examined as a possible attrition routine for one against one minibattles. After altering the model to allow for the possibility of neither side winning due to a line of sight break (the only way a draw could occur previously was if one side jockeyed), the following results were obtained. Figure 11 shows how the probability of neither side winning due to a line of sight break varies with the exposure duration. Figure 12 shows the effect that varying the exposure duration has on the win

probabilities for each side. Similarly, Figure 13 shows the effect that exposure duration has on the probability of a draw due to one side jockeying. These results were derived using the test data set shown in Table 1. This data set is in no way intended to be realistic and is used for comparative purposes only. The exposure duration was decreased from 150 seconds to 50 seconds in 10 second intervals and then from 50 seconds to 10 seconds in 5 second intervals. For each different value, 5000 replications of the model were run.

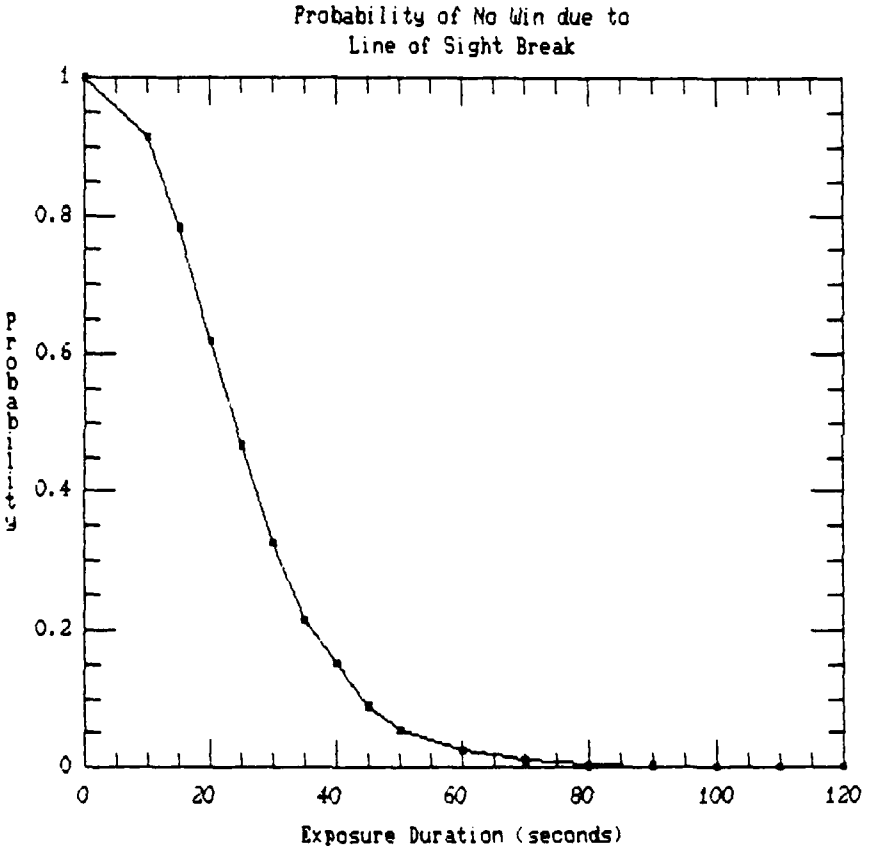


Figure 11

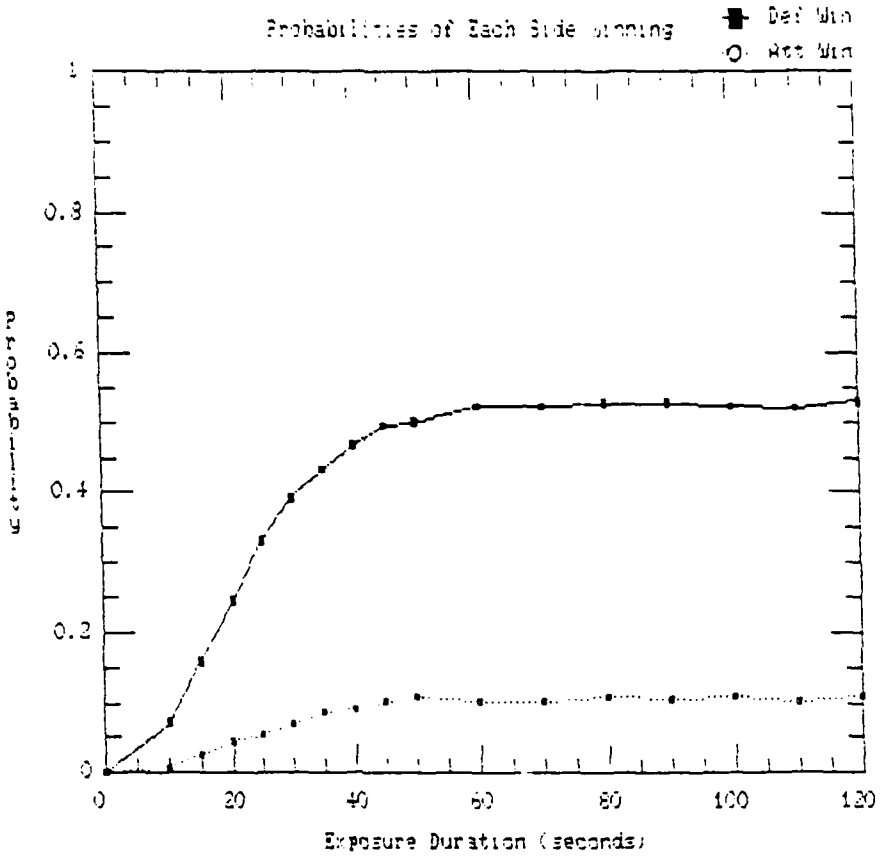

Probabilities of Each Side Winning
Def Win
Att Win
1
0.8
0.6
0.4
0.2
0
0
20
40
60
80
100
120
Exposure Duration (seconds)

Figure 12

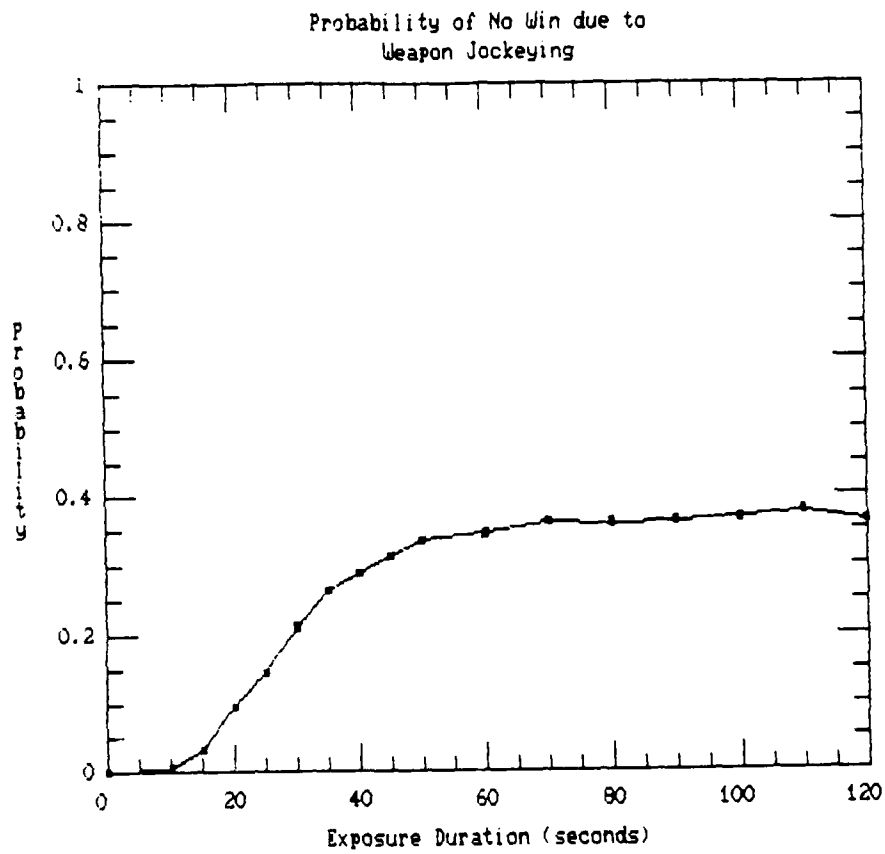

Probability of No Win due to
Weapon Jockeying
Probability
1
0.8
0.6
0.4
0.2
0
0
20
40
60
80
100
120
Exposure Duration (seconds)

Figure 13

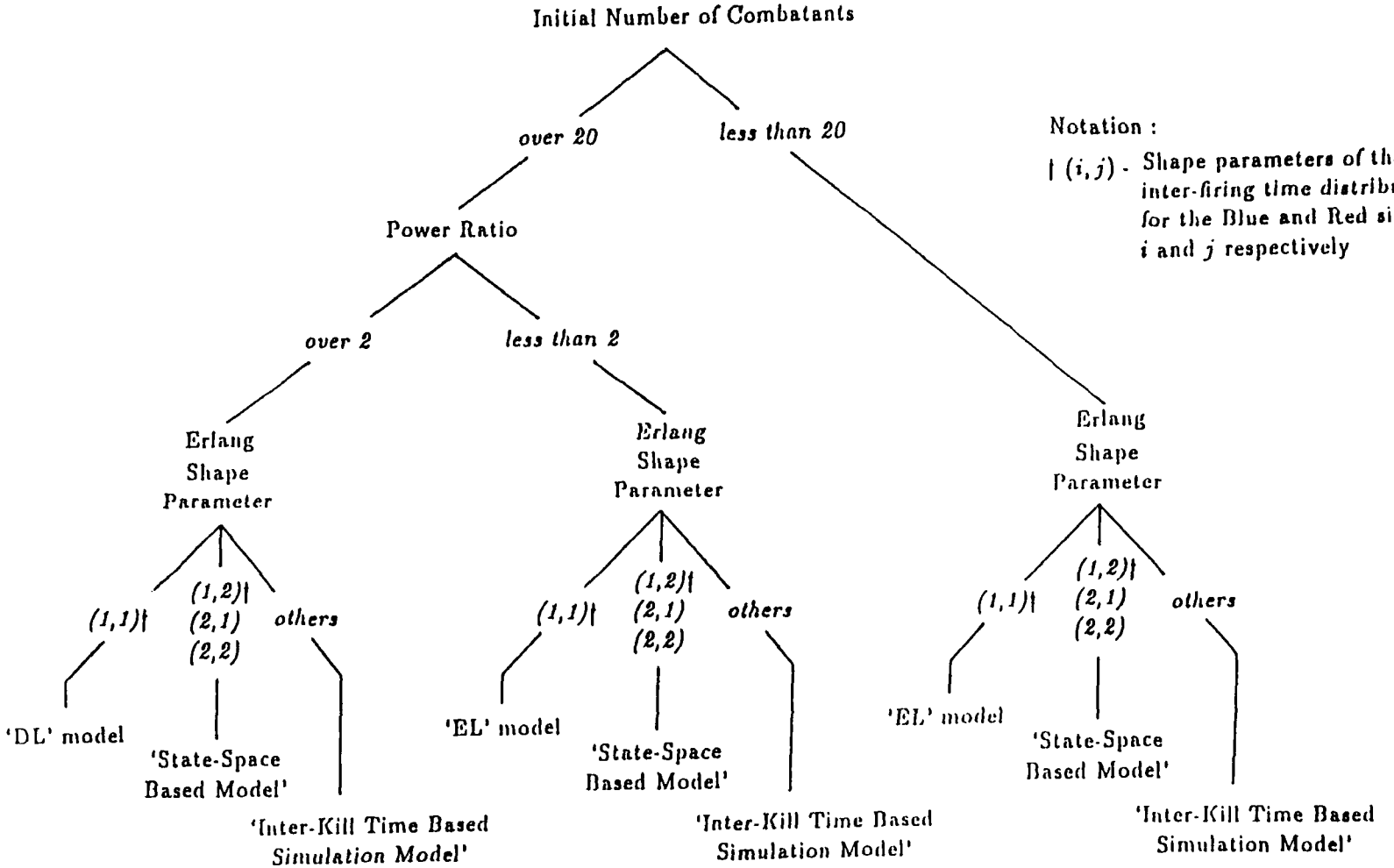
	Attacker	Defender
Mean detection time	12s	100s
Mean time for 1st shot to land after detection	8s	10s
Mean time for subsequent shots to land after 1st	4s	5s
Probability of detecting by firing signature	0.2	0.2
Single shot kill probability	0.2	0.2
Max. No. of shots fired before jockeying	4	4

Table 1

23. For minibattles involving more than two weapon systems, there are various possibilities. The most expensive of these in terms of computing time would be the traditional Monte Carlo simulation. Nonetheless, such a simulation may allow useful comparisons to be made between alternative methodologies.

24. One such methodology involves the generation of inter-kill times and an attrition routine based on this approach is being developed. This will allow any distribution of inter-firing times to be modelled. The differences in results obtained by using one of the Erlang distributions to model inter-firing times rather than the negative exponential distribution inherent in Lanchester methodology, have previously been pointed out by Ancker and Gafarian among others [D,E,F]. Indeed, it may be possible in the future to incorporate in the model some of their work on analytic solutions to the two-on-one and two-on-two duels.

25. A useful guide to the varying suitability of different attrition methodologies for the modelling of combat at different levels is given by Choi [G]. Figure 14 shows a diagrammatic summary of this.



Tree Diagram for the Guidances of Use of the 'DL' Model, the 'EL' Model, the 'State-Space Based Model' and the 'Inter-Kill Time Based Simulation Model'

Figure 14
(reproduced by kind permission of Captain S.Y. Choi from his PhD thesis)

Conclusion

26. Further analysis of data from the ARCOMS trials has been carried out with a view to finding how much time a weapon system spends playing an active role in a battle and how much variation can be expected from weapon to weapon and between attacking and defending forces. Two definitions of activity were considered - the first definition only covering the time spent firing or preparing to fire while the second definition also included the time spent being fired at. The results obtained for the attacker were well fitted by negative exponential distributions while those for the defender were well fitted by Erlang distributions.

27. Average results were also obtained showing how each side's recorded activity level as a proportion of its total possible activity level (calculated by considering all of its survivors) varied as a function of elapsed battle time.

28. A better understanding of the results and of the sometimes large variations in results between battles would be afforded by some descriptive background material on the trials eg. variations in the terrain, tactics, etc. from battle to battle.

29. Development of a prototype model is continuing which will allow a number of ideas to be tested. The forces on each side will have a group structure. The attacking side's groups will travel along a number of attack paths while the defending side's groups will occupy a set of static positions. This will make it easier to identify potential engagements.

30. It is likely that more than one attrition routine will be required as some methodologies are better suited to certain sizes of minibattles than others. Stochastic duel type models are appropriate for very small numbers of combatants and could either be incorporated directly in a model as attrition routines or used as pre-processors providing tables of results to sample from. For larger minibattles, however, an alternative approach is required eg. an inter-kill time based model.

References

A. BATHE, MANWELL, McNAUGHT; 'Modelling Combat as a Series of Minibattles', Final Report to the US Army European Research Office and RMCS Report No. SAG/05/88, November 1989.

B. WAND, HUMBLE, WILSON, DAY; 'MATADOR II - A Model of the Stochastic Tank Duel', RMCS Report No. ORS/03/85, July 1985.

C. WAND; 'Mathematical Models of Military Systems', Ph.D. Thesis, School of Defence Management, RMCS (Cranfield), submitted May 1989.

D. ANCKER, GAFARIAN; 'The Two-on-One Stochastic Duel', Naval Research Logistics Quarterly, Vol 31, pp 309-324.

E. GAFARIAN, MANION; 'Some Two-on-Two Homogeneous Stochastic Combats', Naval Research Logistics Quarterly, Vol 34, pp 505-533.

F. ANCKER, GAFARIAN; 'The Validity of Assumptions Underlying Current Uses of Lanchester's Attrition Rates'. Paper presented to the International Symposium on Advances in Combat Modelling held at RMCS (Cranfield), Shrivenham, September 1984.

G. CHOI; 'Extensions to the Lanchester Combat Modelling Methodology', Ph.D. Thesis, School of Defence Management, RMCS (Cranfield), submitted July 1989.